Hence, adjusted date of initial epoch of minimum is, 1822.25 years, a date 1.1 years before the observed date.

Final equation, $y = 1822.25 + 11.31x$.

TABLE 3.—*Final results*

Epochs of minima observed	Epochs adjusted Calc.	Difference observed —Calc.
1823.3	1822.2	+1.1
33.9	33.6	+0.3
43.5	44.9	−1.4
56.0	56.2	−0.2
67.2	67.5	−0.3
78.9	78.8	+0.1
89.6	90.1	−0.5
1901.7	1901.4	+0.3
13.6	12.8	+0.8
23.9	24.1	−0.2

Great stress is laid by some writers upon the *variability* of the length of the sun-spot cycle, and a great deal of significance is claimed for the variations. In so far as the one hundred years of observations comprised in the above analysis are concerned *there is a very striking constancy of the period* as shown by the small residuals in Table 3, and it is difficult to see any significance to the slight fluctuations which appear.

In writing down the observations in the two columns I and II of Table 2, it is necessary that the last observation should always stand opposite the first and then the others will pair off together.

When the number of observations is odd the middle observation must, of course, stand alone at the foot of either I or II. It also occurs that the weights in this case are always *even* numbers and end at the foot of the table with 0; that is, the middle observation has no weight whatever in fixing the value of b'.

As a final comment we may suggest that it will rarely be necessary to carry out the calculations for a large number of observations, individually, but rather these may be conveniently grouped in two's, three's, five's, etc., thus reducing the large number to a series of, say, 10 or 20 values. A little judicious planning of the layout of problems suffices to bring almost any problem of this kind within the scope of the simple computations in Table 2.

For the sake of completeness we may write here the basic equations which evaluate a and b' following the calculations in Table 2. The computer needs only to follow the simple rules to which these equations lead without necessarily understanding them clearly.

$$a = \frac{\Sigma c}{n} - \frac{n-1}{2} b' \qquad \text{(A)}$$

$$b' = \frac{2\Sigma xc - (n-1)\Sigma c}{N = [2\Sigma x^2 - \frac{n}{2}(n-1)^2]} \qquad \text{(B)}$$

Now the great simplification comes in (B). Expanding the numerator leads to the combination of the observations into pairs, which can be weighted and summed as in the last column of Table 2. The proper weights are $n-1$, $n-3$, $n-5$, etc., in all cases.

Furthermore, the denominator is always a definite number depending only upon how many observations are used. This denominator, N, together with the sum of squares of the natural numbers from 1 to 25 are easily computed, once for all, and are given in Table 4. The sums of squares are really not needed in the present case, but are given as it is sometimes convenient to have them, and tables containing these values are not very numerous.

TABLE 4.—*Values of N and Σn^2 for natural numbers 1 to 25*

$$\left[N = \left[2\Sigma x^2 - \frac{n}{2}(n-1)^2\right],\ x = 0, 1, 2, 3, \text{etc.}\right]$$

n	N	Σn^2
1	--------	1
2	1	5
3	4	14
4	10	30
5	20	55
6	35	91
7	56	140
8	84	204
9	120	285
10	165	385
11	220	506
12	286	650
13	364	819
14	455	1,015
15	560	1,240
16	680	1,496
17	816	1,785
18	969	2,109
19	1,140	2,470
20	1,330	2,870
21	1,540	3,311
22	1,771	3,795
23	2,024	4,324
24	2,300	4,900
25	2,600	5,525

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ON KRICHEWSKY'S METHOD OF FITTING FREQUENCY CURVES

By EDGAR W. WOOLARD

[Weather Bureau, Washington, D. C., March 10, 1924]

A Law of Facility may be described as the approximate expression of the relative frequency with which, in the long run, different values are assumed by a quantity which is dependent on a number of variable items or elements, given certain conditions which seem to be adequately fulfilled in common experience. For example, the Law of Facility in the familar case of the ordinary errors of observation was exhaustively studied many years ago and has long been accurately represented by the so-called Gaussian curve of errors, the equation of which is well known.

In recent years the great value of being able to derive with quantitative precision the curve which shall exhibit the law of facility of a quantity under consideration has come to be realized to a greater and greater degree in an immense variety of fields of study. In any case the problem is to find from a finite number of observations, which give a more or less irregular frequency polygon or histogram, the curve which approximates most closely to the frequency curve which would result if we could have an infinite number of observations.

We now have several well-known methods of fitting curves to observed frequency distributions. The first difficulty in curve fitting is that of choosing a suitable curve from among all the possible algebraic and transcendental curves that suggest themselves; the second difficulty lies in evaluating the constants of the equation of the adopted curve. Until a comparatively recent date, the great majority of applications of the theory of frequency curves were to errors of precision measurements, which, as mentioned above, usually conform closely to the Gaussian or Normal Law. As a result, the Normal Curve became a Procrustean bed to which all possible measurements had to be made to fit; not until late in the nineteenth century did skew curves gain general recognition.[1] Again, it was for a long time taken for granted that the correct method of evaluating the

[1] See Arne Fisher. *The Mathematical Theory of Probabilities.* Vol. I, 2 ed., pp. 178–187. New York, 1922.

constants or parameters of any curve is the method of least squares, although in nine cases out of ten this method turns out to be impracticable.

The two most important systems of frequency curves now in general use are: (1) The Gram-Charlier curves, developed by Gram, Thiele, Edgeworth, and Charlier, which may readily be fitted with the help of the lucid exposition of Fisher [2] and the tables of Jörgensen.[3] (2) The Pearson curves, for which the necessary directions have been excellently set out by Elderton,[4] and the tables gathered together by Pearson.[5] Three methods of evaluating parameters are in use: (1) Method of least squares; (2) Thiele's method of semi-invariants; (3) method of moments.

Other methods of fitting frequency curves have been proposed from time to time,[6] but have not come into extensive use. The most recent method, and one which appears to have many merits, is that of Krichewsky.[7] The author states that his method has been published in order that it may undergo the test of practical experience over a wide range of problems. It is the purpose of the present reviewer to give enough information concerning the method to enable the nonmathematical reader and those to whom the original paper may not be available actually to fit curves by this process.

Those interested only in actually applying the process may omit the following sketch of the theory and pass at once to the practical directions and illustrative example, merely referring to the numbered formulæ when necessary; the notation is explained again in these directions. It may be noted that the Gamma Functions, which appear in the formulæ, have been tabulated, e. g., in Pearson's Tables, and that no knowledge of their theory is required in order to use these tables, beyond the fact that $\Gamma(x+1)=x\Gamma(x)$.

Mathematical theory.—The frequency curve

$$y=f(x), \tag{1}$$

in which x varies between the limits l_1 and l_2, will have as its area up to any given ordinate

$$z=\int_{l_1}^{x} y\,dx=F(x), \tag{2}$$

the total area being

$$a=\int_{l_1}^{l_2} y\,dx. \tag{3}$$

By the Fundamental Theorem of the Integral Calculus

$$\frac{dz}{dx}=y. \tag{4}$$

The type equation which Krichewsky fits to the observed frequency distribution is

$$\frac{dz}{dx}=kz^m(a-z)^n=\phi(z); \tag{5}$$

This equation expresses the conditions that $y=0$ when $z=0$ and when $z=a$, and that y is a maximum for some value of z between 0 and a. After the values of the three parameters k, m, n have been determined, the integration of (5) gives (2), which upon being differentiated gives (1) by virtue of (4).

A wide range of particular algebraic and transcendental curves is covered by (5). If $m=n=0.7864$, the curve is normal; if m and n are between 0 and -1, J-shaped and U-shaped curves result.

The parameters are evaluated by the method of moments; the moments

$$M_r=\int_0^a yz^r dz$$

of the curve (5) are used for this purpose. Putting

$$p=\frac{M_1}{M_0},\quad q=\frac{M_2}{M_1},$$

we find that

$$\begin{aligned} m+1&=\frac{p(a-q)}{a(q-p)},\\ m+n+2&=\frac{a-q}{q-p}, \\ k&=\frac{M_0\Gamma(m+n+2)}{a^{m+n+1}\Gamma(m+1)\Gamma(n+1)},\quad n\ \textit{positive},\\ &=\frac{(aM_0-M_1)\Gamma(m+n+3)}{a^{m+n+2}\Gamma(m+1)\Gamma(n+2)},\quad n\ \textit{negative}. \end{aligned} \tag{6}$$

In virtue of (4), the values of the corresponding moments of the statistics are given by

$$M_0=c\Sigma y^2,\ M_1=c\Sigma y^2z,\ M_2=c\Sigma y^2z^2, \tag{7}$$

c being the class interval.

Equation (5) may be solved by separating the variables, but the integration involved can not in general be performed without the use of series (which do not converge rapidly) unless $m+n$ is a positive integer equal to or greater than 2. (See (5B) below.)

Practical directions for fitting.—1. Prepare the frequency table and histogram in the usual manner. Columns (1) and (2) of Table I give the frequency distribution of rainfall amounts at Washington expressed in the form of ratios to the mean. The successive ordinates of the frequency curve given by observation are those at the midpoints of the classes, at which the class frequencies are assumed to be concentrated [column (3)]; in accordance with custom, divide all the frequencies by the total frequency, and all the class intervals by the class interval, making the total frequency a and the class-interval c each equal to unity; in this way we get the table of abscissae and ordinates given in columns (4) and (5). Column (6) merely assigns consecutive numbers, r, to each observation.

2. Compute the successive areas of the histogram up to each of the r ordinates [column (7)],

$$z_r=\sum_{i=1}^{i=r-1} y_i+\frac{y_r}{2},\quad r=1,2,3\ \cdot\ \cdot\ \cdot\ \cdot \tag{8}$$

3. Compute the quantities given in columns (8)–(11), inclusive; and find the sums of columns (7), (8), (9), (11).

4. The equation which is to be fitted is not that of the frequency curve itself, but

$$\frac{dz}{dx}=kz^m\ (a-z)^n, \tag{5}$$

from which the frequency curve is later derived. The next step is to compute p, q, k, m, n by formulae (6), the quantities M_0, M_1, M_2 being those indicated in Table I.

[2] Arne Fisher, *op. cit.*
[3] N. R. Jörgensen. *Undersögelser over Frequensflader og Korrelation.* Copenhagen, 1916.
[4] W. Palin Elderton. *Frequency-Curves and Correlation.* London, 1906.
[5] Karl Pearson. *Tables for Statisticians and Biometricians.* Cambridge Press, 1914.
[6] See, e. g., J. C. Kapteyn. *Skew Frequency Curves in Biology and Statistics.* Groningen, 1912.
[7] S. Krichewsky. A Method of Curve Fitting. *Ministry of Public Works, Egypt, Physical Department Paper No. 8.* Cairo, 1922.

5. Find $m+n$: (A) If $m+n$ is a positive integer ≥ 2, separate the variables in (5) and integrate; this gives z as a function of x, and upon performing the differentiation (4) we get $y=f(x)$, the equation of the frequency curve. If $m+n$ differs very little from such a positive integer, we may put $n=2-m$ and

$$\frac{m}{2}=\begin{cases}\dfrac{n}{m}, & n<m\\[4pt] \dfrac{m}{n}, & m<n\end{cases}$$

in (5), and integrate.

(B) If $m+n$ is not such a positive integer, choose a suitable number of equally spaced values of z from 0 to 1,[8] and compute the corresponding values of Δx by the approximate finite-difference formula

$$\Delta x=\frac{\Delta z}{\frac{1}{2}(y_r+y_{r+1})}=x_{r+1}-x_r \qquad (9)$$

$$=\frac{z_{r+1}-z_r}{\frac{1}{2}k\{z_r^m(1-z_r)^n+z^m_{r+1}(1-z_{r+1})^n\}}$$

as shown in Table II. Next find

$$d=\Sigma z/a-0.5. \qquad (10)$$

the distance (in class intervals) from the last ordinate of the observed distribution to the ordinate through the mean of this distribution, and calculate the area z_q of the observed distribution up to this ordinate. In general z_q will lie between two of the areas, z_r and z_{r+1}, for which the corresponding theoretical ordinates, y_r and y_{r+1}, and Δx have been computed; compute from (5) y_t the theoretical value of y corresponding to this area, and determine its position relative to y_r by equating $\frac{1}{2}(y_r+y_t)$ δx (the area of the trapezoid formed by joining the tops of y_r, y_t) to z_q-z_r. Take the position thus determined by δx as origin, and with the aid of the Δx's form a table of corresponding values of z, x, and y, as in Table II (x being in terms of class intervals, and y in terms of the total frequency) from which the curve may be plotted; slight irregularities may result from the use of (9).

The area of the fitted curve is equal to the area of the statistics, and the centers of gravity of the two distributions coincide. The mode is given by $z=ma/(m+n)$.

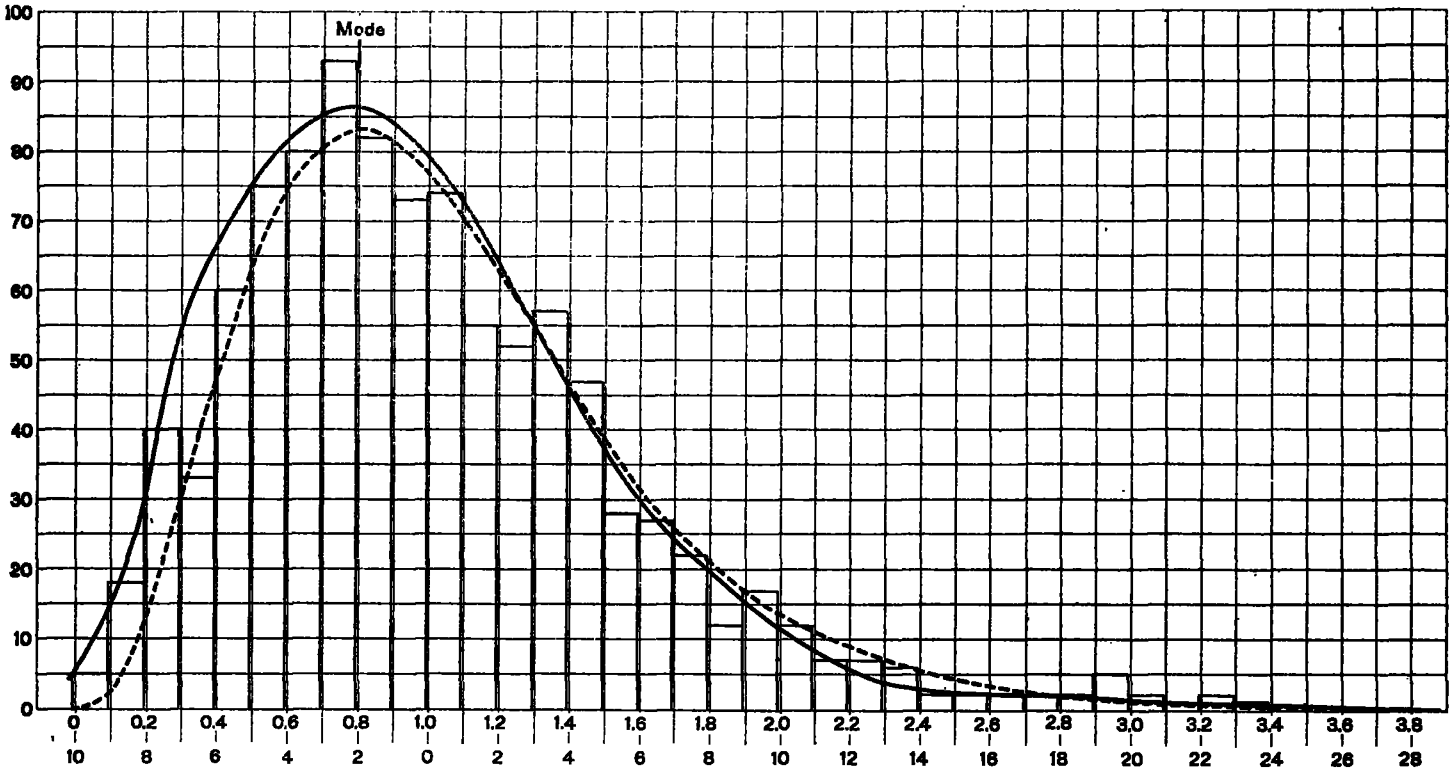

Fig. 1.—Histogram and frequency curve of Washington (D. C.) rainfall data (see Table I). Solid curve: Krichewsky curve of closest fit; dotted curve: Pearson curve (Type III) of closest fit.

In case (B) it is not possible to find an *equation* for the frequency curve itself; but in all cases the ordinates corresponding to any given *areas* can easily be found from (4) and (5). Pearson's system gives the equation to the frequency curve in all cases, but in general these equations cannot be integrated; and areas, which are often what is really desired, cannot be found without great labor. The arithmetical work in fitting the curve is much more laborious in Pearson's system than by the present method. The resulting fits by the two methods appear to be equally good; a number of examples are given by Krichewsky. The best-fitting Pearson curve for the data in Table I is that of Type III given by the equation—

$$y=834.3\left(1+\frac{x}{0.782}\right)^{2.835}\exp\left(-3.626x\right).$$

The accompanying Figure 1 shows both curves and the histogram.

[8] In order to carry the fit out to the limits of the observed distribution it is necessary to choose these values much closer together near 0 and 1 than in the middle.

TABLE I

Class (1)	Freq. (2)	Mid-points (3)	x (4)	y (5)	r (6)	z (7)	y^2 $\times 10^2$ (8)	y^2z $\times 10^2$ (9)	z^2 (10)	y^2z^2 $\times 10^2$ (11)
0.0–0.1	5	0.05	0.5	0.005	1	0.0025	0.0025	0.00000	0.0000062	0.00000
0.1–0.2	18	0.15	1.5	.018	2	.0140	.0324	.00045	.0001960	.00000
0.2–0.3	40	0.25	2.5	.040	3	.0430	.1600	.00688	.0018490	.00029
0.3–0.4	33	0.35	3.5	.033	4	.0795	.1089	.00865	.0063202	.00069
0.4–0.5	60	0.45	4.5	.060	5	.1260	.3600	.04536	.0158760	.00572
0.5–0.6	75	0.55	5.5	.075	6	.1935	.5625	.10884	.0374422	.02106
0.6–0.7	80	0.65	6.5	.080	7	.2710	.6400	.17344	.0734410	.04700
0.7–0.8	93	0.75	7.5	.093	8	.3575	.8649	.30920	.1278062	.11054
0.8–0.9	82	0.85	8.5	.082	9	.4450	.6724	.29922	.1980250	.13315
0.9–1.0	73	0.95	9.5	.073	10	.5225	.5329	.27844	.2730062	.14548
1.0–1.1	74	1.05	10.5	.074	11	.5960	.5476	.32637	.3552160	.19452
1.1–1.2	55	1.15	11.5	.055	12	.6605	.3025	.19980	.4362602	.13197
1.2–1.3	52	1.25	12.5	.052	13	.7140	.2704	.19306	.5097960	.13785
1.3–1.4	57	1.35	13.5	.057	14	.7685	.3249	.24968	.5905922	19188
1.4–1.5	47	1.45	14.5	.047	15	.8205	.2209	.18125	.6732202	.14871
1.5–1.6	28	1.55	15.5	.028	16	.8580	.0784	.06727	.7361640	.05772
1.6–1.7	27	1.65	16.5	.027	17	.8855	.0729	.06455	.7841102	.05716
1.7–1.8	22	1.75	17.5	.022	18	.9100	.0484	.04404	.8281000	.04008
1.8–1.9	12	1.85	18.5	.012	19	.9270	.0144	.01335	.8593290	.01237
1.9–2.0	17	1.95	19.5	.017	20	.9415	.0289	.02721	.8864222	.02562
2.0–2.1	12	2.05	20.5	.012	21	.9560	.0144	.01377	.9139360	.01316
2.1–2.2	7	2.15	21.5	.007	22	.9655	.0049	.00473	.9321902	.00457
2.2–2.3	7	2.25	22.5	.007	23	.9725	.0049	.00476	.9457562	.00463
2.3–2.4	6	2.35	23.5	.006	24	.9790	.0036	.00352	.9584410	.00345
2.4–2.5	2	2.45	24.5	.002	25	.9830	.0004	.00039	.9662890	.00039
2.5–2.6	2	2.55	25.5	.002	26	.9850	.0004	.00039	.9702250	.00039
2.6–2.7	0	2.65	26.5	.000	27	.9860	.0000	.00000	.9721960	.00000
2.7–2.8	2	2.75	27.5	.002	28	.9870	.0004	.00039	.9741690	.00039
2.8–2.9	2	2.85	28.5	.002	29	.9890	.0004	.00039	.9781210	.00039
2.9–3.0	5	2.95	29.5	.005	30	.9925	.0025	.00248	.9850562	.00246
3.0–3.1	2	3.05	30.5	.002	31	.9960	.0004	.00040	.9920160	.00040
3.1–3.2	0	3.15	31.5	.000	32	.9970	.0000	.00000	.9940090	.00000
3.2–3.3	2	3.25	32.5	.002	33	.9980	.0004	.00040	.9960040	.00040
3.3–3.4	1	3.35	33.5	.001	34	.9995	.0001	.00010	.9990002	.00010
Sums						23.9220	5.8786 M_o	2.62878 M_1		1.48254 M_2

$p=\frac{M_1}{M_o}=0.4472$; $q=\frac{M_2}{M_1}=0.5640$; $m=0.6695$; $n=1.0634$; $k=0.2743$; $m+n=1.7329$.

TABLE II

z (1)	k (2)	$1-z$ (3)	$\log z$ (4)	$\log (1-z)$ (5)	$m \log z$ (6)	$n \log (1-z)$ (7)	$\log y$ (8)	$y\times 10$ (9)	$\frac{1}{2}(y_k+y_{k+1})\times 10$ (10)	Δx (11)	x (12)
0.000	0	1.000		0.00000		0.000000					
.001	1	.999	7.00000	9.99956	4.686500	10.633532	7.42926	0.02687	0.0348	0.28736	
.002	2	.998	7.30103	9.99913	4.888039	10.633075	7.63034	.04269	.0552	.36232	10.17562
.004	3	.996	7.60206	9.99826	5.089579	10.632150	7.83096	.06776	.0782	.25575	9.88826
.006	4	.994	7.77815	9.99739	5.207471	10.631224	7.94792	.08870	.1065	.37559	9.52594
.010	5	.990	8.00000	9.99564	5.356000	10.629364	8.09459	.12434	.1600	.62500	9.27181
.020	6	.980	8.30103	9.99123	5.557539	10.624674	8.29144	.19563	.2247	.44504	8.89622
.030	7	.970	8.47712	9.98677	5.675432	10.619931	8.40459	.25386	.2791	.35829	8.27122
.040	8	.960	8.60206	9.98227	5.759079	10.615146	8.48345	.30440	.3270	.30581	7.82618
.050	9	.950	8.69897	9.97772	5.823960	10.610307	8.54350	.34954	.4372	1.14364	7.46789
.100	10	.900	9.00000	9.95424	6.025500	10.585339	8.72006	.52488	.5864	0.85266	7.16208
.150	11	.850	9.17609	9.92942	6.143392	10.558945	8.81156	.64798	.6923	.72223	6.01844
.200	12	.800	9.30103	9.90309	6.227040	10.530046	8.86721	.73656	.7675	.65146	5.16578
.250	13	.750	9.39794	9.87506	6.291921	10.501139	8.90228	.79851	.8184	.61095	4.44355
.300	14	.700	9.47712	9.84510	6.344932	10.469279	8.92344	.83838	.8487	.58914	3.18114
.350	15	.650	9.54407	9.81291	6.389755	10.435048	8.93403	.85907	.8609	.58079	2.59200
.400	16	.600	9.60206	9.77815	6.428579	10.398085	8.93589	.86276	.8569	.58350	2.01121
.450	17	.550	9.65321	9.74036	6.462824	10.357899	8.92995	.85104	.8381	.59659	1.42771
.500	18	.500	9.69897	9.69897	6.493460	10.313885	8.91657	.82522	.8058	.62050	0.83112
.550	19	.450	9.74036	9.65321	6.521171	10.265223	8.89562	.78636	.7609	.65712	0.26062
.600	20	.400	9.77815	9.60206	6.546471	10.210831	8.86653	.73541	.7043	.70992	0.86774
.650	21	.350	9.81291	9.54407	6.569743	10.149164	8.82813	.67318	.6368	.78518	1.57766
.700	22	.300	9.84510	9.47712	6.591294	10.077969	8.77849	.60047	.5592	.89413	2.36284
.750	23	.250	9.87506	9.39794	6.611353	9.993769	8.71435	.51802	.4723	1.05865	3.25697
.800	24	.200	9.90309	9.30103	6.630119	9.890715	8.63006	.42604	.3769	1.32661	4.31562
.850	25	.150	9.92942	9.17609	6.647747	9.757854	8.51483	.32721	.2741	1.82415	5.64223
.900	26	.100	9.95424	9.00000	6.664364	9.570600	8.34419	.22090	.1652	3.02663	7.46638
.950	27	.050	9.97772	8.69897	6.680084	9.250485	8.03980	.10960	.0870	2.29885	10.49301
.970	28	.030	9.98677	8.47712	6.686142	9.014569	7.80994	.06456	.0424	4.71698	12.79186
.990	29	.010	9.99564	8.00000	6.692081	8.507200	7.30851	.02035	.0151	3.31126	17.50884
.995	30	.005	9.99782	7.69897	6.693540	8.187085	6.98998	.00977	.0077	2.59710	20.82010
.997	31	.003	9.99869	7.47712	6.694123	7.951169	6.75452	.00568	.0037	5.40541	23.41720
.999	32	.001	9.99956	7.00000	6.694705	7.443800	6.24773	.00177			28.82261
1.000	33	.000	0.00000		0.000000						

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COMMENTS ON THE LAW OF PRESSURE RATIOS

By F. J. W. WHIPPLE

[6 Addison Road, Chiswick, London, W 4, January 2, 1924]

In his paper on "The Law of Pressure Ratios and its Application to the Charting of Isobars in the Lower Levels of the Troposphere," Dr. C. Le. Roy Meisinger has reached conclusions to which he has given some prominence but which seem to be based on insufficient evidence. The object of this letter is to point out that the argument by which Doctor Meisinger shows that there is a functional relation between his variables x and y shows also that there is an upper limit to the constant which he calls a.

It is convenient to make a small change from Meisinger's notation and write T_{oz} for the average value of the absolute temperature between the heights s and z.

By definition y is the ratio of pressures at the heights z and s kilometers above sea level so that $y=\exp(-zc/T_{sz})$, where c is a constant.

If T'_{sz} is the mean value of T_{sz} and ΔT_{sz} the departure from the mean, then,

$$y=[1+zc\Delta T_{sz}/T'^2_{sz}]\exp(-zc/T'_{sz}).$$

Similarly, x, the ratio of pressures at the heights 1 and 2 kilometers may be expressed as follows:

$$x=[1+c\Delta T'^2_{12}/T'^2_{12}]\exp(-c/T'_{12}).$$

Now the regression equation by which x and y are associated may be written

$$y=ax+b+\epsilon.$$

Here a and b are Meisinger's constants and ϵ is a residual varying term which is not correlated with x. The coefficient a can be found by the method of least squares; it is given by the equation

$$a=z\exp[(c/T'_{12})-(c_z/T'_{sz})]\,(T'_{12}/T'_{sz})^2\,(\sigma_{sz}/\sigma_{12})\,r_{sz.12}$$

In this equation, σ_{sz} and σ_{12} are the standard deviations of T_{sz} and T_{12} respectively, whilst $r_{sz.12}$ is the correlation coefficient for those two variables.

In discussing the possible values of a it will suffice for our present purpose to confine attention to the case in which s and z are identical with 0 and 3, respectively. In this case, T_{12} and T_{sz} are the mean temperatures of columns both centered $1\frac{1}{2}$ kilometers above ground. These two quantities will differ by very little from one another in ordinary circumstances and we may write as very good approximations:

$$T'_{12}=T'_{03}$$
$$\sigma_{12}=\sigma_{03}$$
$$A=3\exp(-2c/T'_{12})\,r_{03.12}$$

For T'_{12} we may take the annual mean for the United States[1] $T'_{12}=279$

It follows[2] that $\exp(-c/T'_{12})=:8847$ and hence that

$$A=3\times :8847^2\,r_{03.12}=2.35\,r_{03.12}$$

Now the correlation coefficient must be near to unity but it can not exceed unity. Hence 2.34 is the upper limit for the coefficient a for 3 kilometers.

The values obtained by Doctor Meisinger at two of his stations are 2.58 and 2.76, respectively. These figures seem to be too high; they could only be justified by the supposition that σ_{03} exceeded σ_{12} considerably. This might happen if the series of observations included a large number of "inversions" of temperature but the available evidence is against this supposition. The tables for the stations in question, Groesbeck and Leesburg in Gregg's "Aerological Survey of the United States" do not show any excessive frequency of cold air at the surface and moreover the observations which were utilized both by Gregg and by Meisinger were

[1] Gregg: *Aerological Survey of the United States.* MO. WEATHER REV. SUPP. No. 20, Table 6.
[2] *Computer's Handbook,* London, 1917. 11.2.44.